AD-763 814

CURVATURE SPECTRA: APPLICATIONS TO SIGNAL DETECTION AND ESTIMATION OF THE RESOLVING POWER OF ARRAYS

D. W. McCowan, et al

Teledyne Geotech

Prepared for:

Air Force Office of Scientific Research
Advanced Research Projects Agency

March 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

AL-72-7

.........contributing to man's understanding of the environment world

CURVATURE SPECTRA: APPLICATIONS TO SIGNAL DETECTION AND ESTIMATION OF THE RESOLVING POWER OF ARRAYS

BY

D.W. McCOWAN and E.A. FLINN

SPONSORED BY
ADVANCED RESEARCH PROJECTS AGENCY

MONITORED BY
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

ARPA ORDER NO. 1357

23 MARCH 1973

TELEDYNE GEOTECH

ALEXANDRIA LABORATORIES

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Teledyne Geotech Alexandria, Virginia 22313	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP
3. REPORT TITLE Curvature Spectra: Applications to Signal Detection and Estimation of the Resolving Power of Arrays	
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Scientific Interim	
5. AUTHOR(S) (First name, middle initial, last name) D W McCowan, E A Flinn	
6. REPORT DATE Mar 1973	7a. TOTAL NO. OF PAGES ~~36~~ 44 — 7b. NO. OF REFS 7
8a. CONTRACT OR GRANT NO. F44620-69-C-0082 b. PROJECT NO. A01357 c. 62701D d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AFOSR - TR - 73 - 1144
10. DISTRIBUTION STATEMENT Approved for Public Release; Distribution unlimited.	
11. SUPPLEMENTARY NOTES TECH, OTHER	12. SPONSORING MILITARY ACTIVITY AFOSR (NPG) 1400 Wilson Boulevard Arlington, Virginia 22209

13. ABSTRACT

Curvature spectra convey roughly the same information as ordinary or high-resolution f-k spectra, with resolution intermediate between the two. The curvature of an array response fuction contains information about an array not supplied in the array response fuction itself. For example, the polarity of side lobes, i.e., whether they are small peaks or valleys, is better judged from curvature response plots. The differential response of an array provides a means of comparing the angular resolution of different arrays having the same signal detection sensitivity. Azimuth resolution of the infrasonic analog correlator detector is 0.95 mb. At 20 sec. period and 200 m/sec phase velocity, experience shows that the angular resolution of the Huancayo microbarograph array is about one degree of azimuth, which implies that theoretically the angular resolution of LAMA, measured by the same technique is about one minute of arc.

DD FORM 1 NOV 65 1473

iv

UNCLASSIFIED

Security Classification

CURVATURE SPECTRA: APPLICATIONS TO SIGNAL DETECTION AND ESTIMATION OF THE RESOLVING POWER OF ARRAYS

ALEXANDRIA LABORATORIES REPORT NO. AL-72-7

Effective Date of Contract:	1 March 1969
Contract Expiration Date:	31 March 1972
Amount of Contract Dollars:	$ 590,572
Program Code:	1F10
Contract Number:	F-44620-69-C-0082
ARPA Order No.:	1357
Principal Investigator:	Carl A. Newton
Period Covered:	1 March 1969 through 31 March 1972

ABSTRACT

We define the curvature spectrum as the two-dimensional Laplacian of a frequency-wavenumber spectrum, the derivatives being taken in the wavenumber directions. Results show that the method has the characteristics of a high-resolution f-k estimator, i.e., the spectral peaks are sharper than in ordinary f-k spectra. The curvature of an array response function can give additional information about the array capability, in addition to the array response function itself. In particular, defining the <u>differential array response</u> as the curvature of the array response at $\underset{\sim}{k} = 0$, we show that this differential response predicts the angular resolution capability of an array. We conclude that if two arrays have differential responses D_1 and D_2, then their angular resolution capabilities θ_1 and θ_2 are related by $D_1^{1/2}\,\theta_1 = D_2^{1/2}\,\theta_2$. It is shown that using the N-4 correlator signal analysis algorithm, the azimuth resolution of four- and five-element arrays with apertures as small as 20 km is about one-tenth of a degree. The capabilities of some microbarograph arrays in current use are compared.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (Cont'd.)

INTRODUCTION

An important consideration in the design of arrays is the resolution capability for determining signal velocity and arrival azimuth, given a particular configuration of elements. This is conventionally defined as the width of the central peak of the array response function in wavenumber space at a specified number of decibels down from the peak gain. For example, the width of the central peak of the 13-element LAMA microbarograph array (Figure 1) at -3 db is approximately 0.02 units of wavenumber. In this report, we will always choose k to be inverse wavelength, so its units are km^{-1}.

It is apparent that such a specification is imprecise, since it depends on an arbitrary gain level, and when used for detection analysis, also on the sensitivity and resolution of the detection algorithm. Furthermore, it tells nothing about the shape of the central peak above the chosen gain level. It is conceivable that two different array configurations could give array response functions having the same width of the central peak at a certain gain level but having radically different shapes, as illustrated schematically in Figure 3.

In this study we propose another measure of array resolution capability differential quantity that depends

only on the shape of the central peak at its maximum height, and compare this with other standard measures. We also present data indicating that the detection sensitivity of the N4 multichannel correlator algorithm (Brown, 1962; Smart and Flinn, 1971) is of the order of millibels, not decibels, for the four- and five-element ESSA microbarograph arrays.

THEORETICAL DISCUSSION

The usual expression for frequency-wavenumber spectra (of which the array response function is a special case) is:

$$P(\omega,\underset{\sim}{k}) = \sum_{i=1}^{N} \sum_{j=1}^{N} S_{ij}(\omega)\exp[i\underset{\sim}{k}\cdot(\underset{\sim}{x}_i-\underset{\sim}{x}_j)] \tag{1}$$

Our method is to compute the two-dimensional Laplacian:

$$\nabla_k^2 = \nabla_k\cdot\nabla_k \tag{2}$$

of the f-k spectrum in wavenumber space:

$$\nabla_k^2 P(\omega,\underset{\sim}{k}) = -\sum_{i=1}^{N} \sum_{j=1}^{N} S_{ij}(\omega)|\underset{\sim}{x}_i-\underset{\sim}{x}_j|^2\exp[i\underset{\sim}{k}\cdot(\underset{\sim}{x}_i-\underset{\sim}{x}_j)] \tag{3}$$

This quantity is also a function of ω and $\underset{\sim}{k}$, which is related to the shape of curvature of the f-k spectrum surface.

When applied to the array response function

$$A(\underset{\sim}{k}) = \sum_{i=1}^{N} \sum_{j=1}^{N} \exp[i\underset{\sim}{k}\cdot(\underset{\sim}{x}_i-\underset{\sim}{x}_j)] \tag{4}$$

and evaluated at the origin (i.e., the point $\underset{\sim}{k} = 0$), this is:

$$D(\underset{\sim}{k})=\nabla_k^2 A(\underset{\sim}{k}) = -\sum_{i=1}^{N} \sum_{j=1}^{N} |\underset{\sim}{x}_i-\underset{\sim}{x}_j|^2 \tag{5}$$

We define this as the <u>differential array response</u>, since it is not a function of $\underset{\sim}{k}$, but depends only upon the coordinates of the array elements.

For cylindrically symmetric surfaces, i.e., surfaces which are symmetric about the frequency axis $\underset{\sim}{k} = 0$, the two-dimensional Laplacian at the peak is also related to the radius of curvature of a sphere passing through the surface at that point. It is because of this relation that we use the term "curvature spectra".

RESULTS

The results of this study are given in Table III and the figures. As an example of real data, we chose the acoustic-gravity wave signals generated by an atmospheric nuclear explosion, recorded by the array of ESSA micro-barographs at LAMA. Information about the signal is given in Table I; Table II gives the coordinates of the micro-barograph array elements. For a description of instrumentation, see Cook and Bedard (1971).

These contour plots presented here were computed by the method described by McCowan (1968). The functions plotted are described in the figure captions. For simplicity of comparison with the other spectra, the negative of the curvature spectra are plotted.

Figure 4 shows the frequency-wavenumber spectra of the acoustic-gravity wave signal at four periods; Figure 5 shows high-resolution f-k spectra (McCowan and Lintz, 1968) at the same four periods; and Figure 6 shows curvature spectra, again at the same four periods.

Figure 2 shows the curvature array response of the thirteen innermost elements of LAMA, over the same wavenumber range as the ordinary array response in Figure 1. It is apparent that the central peak is narrower by about half in the curvature response. Thus

we should expect a "high-resolution" effect (McCowan and Lintz, 1968) in the curvature spectra. In addition, the curvature response enables us to see more detail in the regular array response. For example, the ears of the -9 db contour in the regular response appear as small side lobes in the curvature response.

The high-resolution spectra were computed with a signal-to-noise ratio parameter 0.01 (McCowan and Lintz, 1968).

Comparing these results, we see that the curvature spectra show a resolution greater than the ordinary but not as high as the high-resolution f-k spectra. It appears that at 128 seconds period the high-resolution method resolves two peaks, where neither of the other two methods resolves more than one peak.

Figures 7, 8, and 9 show the ordinary Huancayo array response contoured in decibels, in centibels (equal 0.1 to decibel), and millibels (equal to 0.01 decibel). It is significant to note that as the contour levels become closer and closer to the peak value (Figure 9), the central peak becomes nearly circular. Thus it appears that we are justified in representing the peak of the central lobe of an array response as a spherical surface in the region around the peak value.

Array resolution capability

Figure 10 shows a segment of output from our N-4 correlator program using as input an artificial set of records containing a monochromatic plane wave with period 20 seconds, velocity 300 m/sec, and arrival azimuth 180°, recorded at the innermost five elements of the LAMA array. Four beams were used, each with velocity 300 m/sec, and with azimuths spaced one-half degree away from the known arrival azimuth. It is obvious that this array, whose effective aperture would be about 30 km, judging from the half-power points of the array response, is theoretically capable of resolving azimuth shifts of about one-tenth of a degree, to take a conservative lower limit. This result is consistent with results obtained using the analog version of the N-4 correlator on four- and five-element ESSA microbarograph arrays. We note, however, that this resolution is attained on the analog version of the correlator only by "hand-setting" the device, since in the normal continuous beamsweeping generating mode there is a smearing of azimuth due to the fact that the beam direction moves while a given segment of data is in the tape loop (H. Matheson, personal communication, 1969). The digital version of the correlator does not suffer from this drawback, but clearly

because of the finite sampling interval there is a minimum azimuth resolution for a given size of the array (see below).

The period and phase velocity 20 seconds and 300 m/sec correspond to a region of f-k space centered at wavenumber

$$|k| = \frac{1}{cT} = \frac{1}{6} \approx 0.167 \text{ km}^{-1} \tag{10}$$

The width of a region in wavenumber space centered at this wavenumber and intercepting one degree of azimuth is:

$$k' = k\theta = \frac{1}{cT} \cdot \frac{\pi}{180} \approx 2.9 \cdot 10^{-3} \tag{11}$$

Referring to Figure 9, we can see that this represents a difference in gain of less than one millibel at the Huancayo array. A more precise measurement gives the results 0.88 millibel in the east-west direction and 0.99 millibel in the north-south direction. Thus we are able to say that a human operator working with the analog correlator under these conditions can probably distinguish gains which differ by about 0.95 millibel.

We pointed out that the two-dimensional Laplacian of a spherical surface is inversely proportional to its radius of curvature. For a spherical surface the radius of curvature is proportional to the width, so we can say that if two arrays have differential responses D_1 and D_2, then their angularular resolution is related by:

$$D_1^{1/2}\theta_1 = D_2^{1/2}\theta_2 \tag{12}$$

Using equation (12) and knowing the angular resolution of an array which uses some given detection algorithm, we can calculate the angular resolution of any other array under the same conditions using the same detection algorithm. This is done in Table III, taking 0.1 degree for the five-element LAMA array as a standard. An analog correlator processing the thirteen-element LAMA array could theoretically resolve arrival azimuths to within about one minute of arc.

This analysis is true only for the central peak of the array response. If the array is mis-steered badly enough so that a sidelobe peak is being seen, then the results are obviously inapplicable. This is an unfortunate consequence of a differential quantity: it tells nothing about the other peaks. However, we will assume, as those

who operate the analog correlator do, that we know in advance where in wavenumber space to expect infrasonic signals, i.e., at sonic velocities and wavenumber. In practice this difficulty arises only at the shorter periods. This theoretical capability is also obviously degraded if the signal waveform changes across the width of the array, or if there is significant noise in the data.

Effect of sampling rate on beaming accuracy

A finite sampling rate imposes a limit on azimuthal resolution: for a given arrival direction, we obviously cannot tell the difference between two beams so close together that the two sets of beam shift indices are the same integers. To investigate whether this is a limitation on the digital N-4 correlator processor operating on arrays of the size of the present microbarograph arrays, we used our artificial monochromatic plane wave data, and searched over an azimuth grid so fine that the beam shift integers differed from one another by only one integer at one of the five channels. The azimuth intervals were 0.5°. For example, the beam toward 178.0° had shift integers 45, 0, -19, 59, -40; the beam toward 178.5° had shift integers 45, 0, -20, 59, -40. Figure 19

shows a comparison of two outputs of these runs, one for a sampling rate of 4 Hz and one for a sampling rate of 1000 Hz. It is clear that at sampling rates at least as low as 4 Hz the finite sampling interval does not degrade the correlator output.

CONCLUSIONS

Curvature spectra convey roughly the same information as ordinary or high-resolution f-k spectra, with resolution intermediate between the two.

The curvature of an array response function contains information about an array not supplied in the array response function itself. For example, the polarity of side lobes, i.e., whether they are small peaks or valleys, is better judged from curvature response plots.

The differential response of an array provides a means of comparing the angular resolution of different arrays having the same signal detection sensitivity. Azimuth resolution of the infrasonic analog correlator detector is 0.95 mb. At 20 sec. period and 200 m/sec phase velocity, experience shows that the angular resolution of the Huancayo microbarograph array is about one degree of azimuth, which implies that theoretically the angular resolution of LAMA, measured by the same technique is about one minute of arc.

ACKNOWLEDGEMENTS

This research was supported by the Advanced Research Projects agency under Project VELA HOTEL, and monitored by the Air Force Office of Scientific Research under Contract F44620-69-C-0082.

REFERENCES

Brown, R.F., 1962, An automatic multichannel correlator: J. Res. NBS, v. 67C, p. 33-38.

Cook, R.K. and Bedard, A.J., (in press), On the measurement of infrasound: Geophys. J. Roy. Astr. Soc.

McCowan, D.W., 1968, Design and evaluation of certain multichannel filters: Seismic Data Laboratory Report No. 209, Teledyne Geotech, Alexandria, Virginia.

McCowan, D.W., 1968, Digital computer programs for the design and evaluation of multichannel filters: Seismic Data Laboratory Report No. 210, Teledyne Geotech, Alexandria, Virginia.

McCowan, D.W. and Lintz, P.R., 1968, High-resolution frequency-wavenumber spectra: Seismic Data Laboratory Report No. 206, Teledyne Geotech, Alexandria, Virginia.

Morse, P.M. and Feshbach, H., 1953, Methods of theoretical physics: McGraw Hill Book Co., Inc., New York.

Smart, E. and Flinn, E.A., 1971 (in press), Fast frequency-wavenumber analysis and Fisher signal detection in real-time infrasonic array data processing: Geophys. J. Roy Astr. Soc.

TABLE I*

Event Data for the Acoustic-Gravity Wave Signal

Array	LAMA
Number of elements	13
Sampling rate	2.0 samples/sec
Time window	02:25:44 - 20:34:16 GMT (25 August 1968)
Group velocity window	394 - 384 m/sec
Date	24 August 1968
Latitude	22S
Longitude	139W
Seismic magnitude (m_b)	5.0
Epicentral distance	8300 km
Azimuth of epicenter from array	211° east of north

*Based on information taken from the NOAA/NOS Preliminary Determination of Epicenters releases.

TABLE II

Array Element Coordinates

LAMA

Element	X Coordinate (km)	y Coordinate (km)
B1	9.98	7.08
A0	0.00	0.00
B3	-7.25	-3.27
C4	-11.59	5.20
B4	-1.59	8.82
C1	7.23	16.77
C2	16.05	-2.11
B2	4.55	-5.96
C3	-2.11	-12.72
D3	-19.79	-15.41
D4	-12.27	28.12
D1	23.39	16.85
D2	16.23	-20.61

Hyancayo

Element	x Coordinate (km)	y Coordinate (km)
C7	0.00	0.00
C9	2.48	-4.12
C11	7.43	-2.08
C13	4.36	3.65

Table III

Differential Responses for Several Microbarograph Arrays

Array	Number of Elements	Differential Response (square km)	Predicted Angular Resolution* (degrees)
Peñas	4	-497	0.26
Huancayo	4	-500	0.26
LAMA (inner thirteen elements)	13	-1.11×10^{5}	0.018
Washington	5	-1520	0.15
Washington (without DALE)	4	-1050	0.18
LAMA (inner five elements)	5	-3402	0.1 (assumed

*Assuming a 300 m/sec wave at 20 sec period.

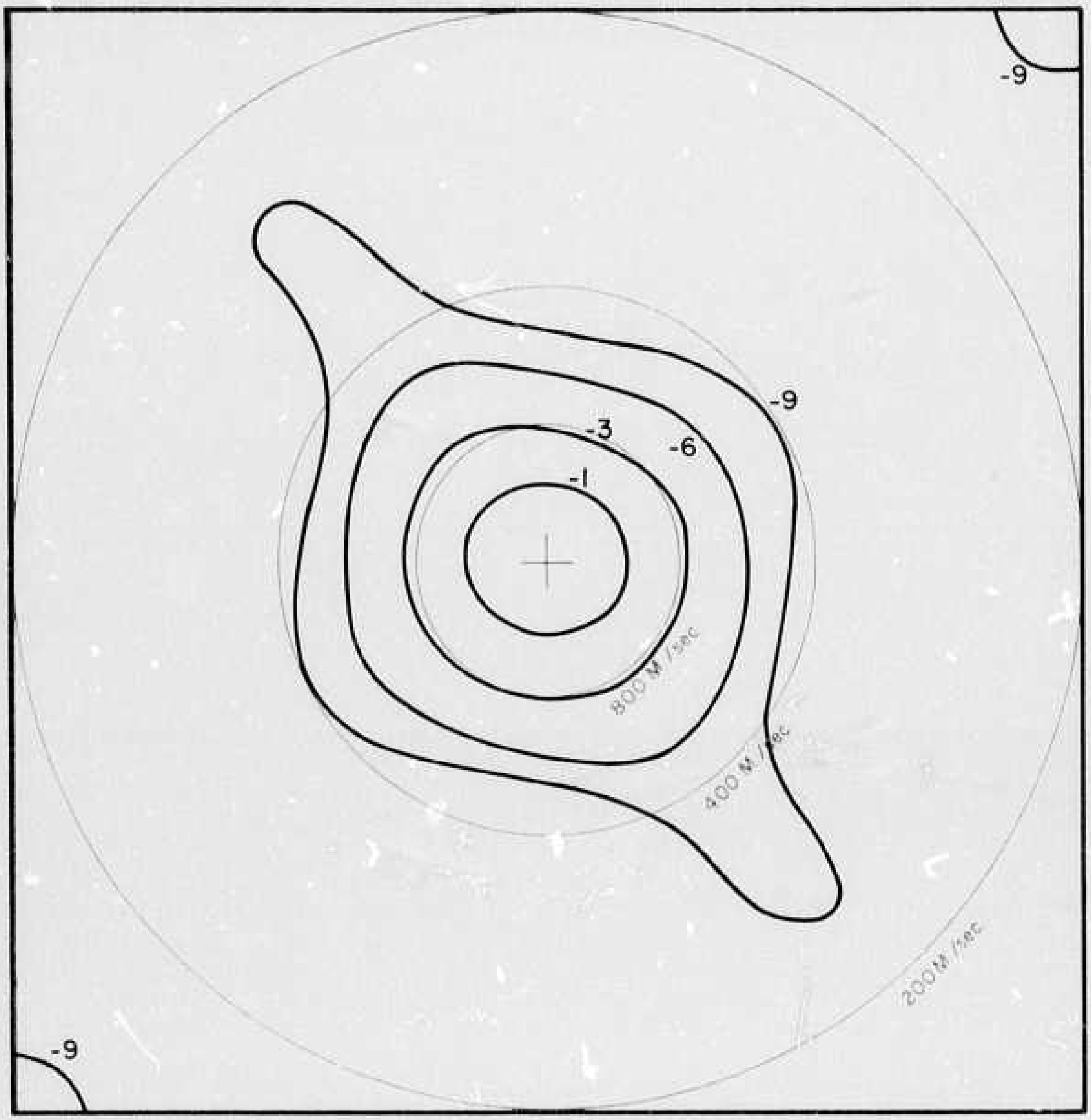


Figure 1. LAMA array response; contours in db. Velocity circles are shown for 128 seconds period.

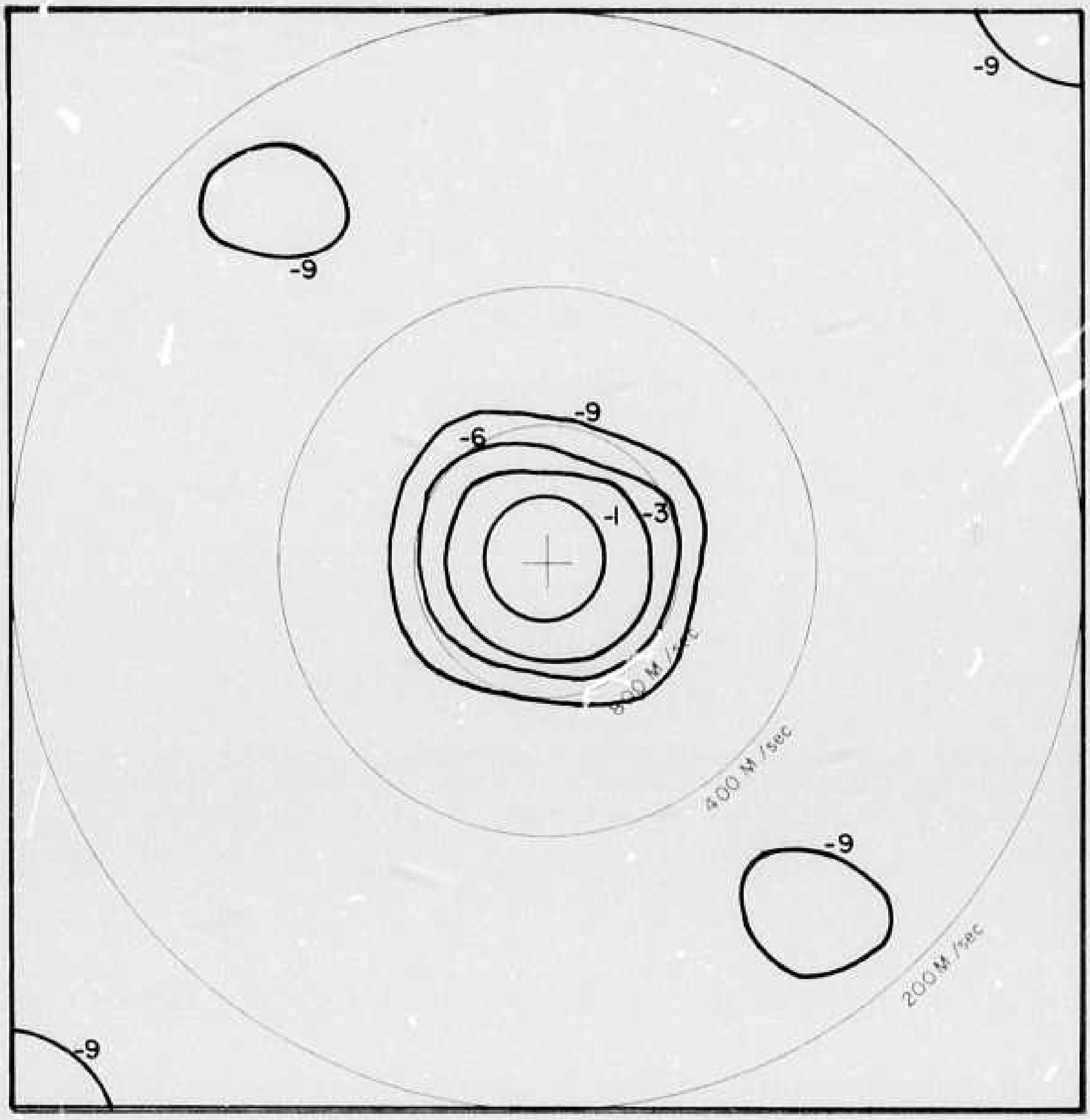


Figure 2. LAMA curvature response; velocity circles for 128 seconds period.

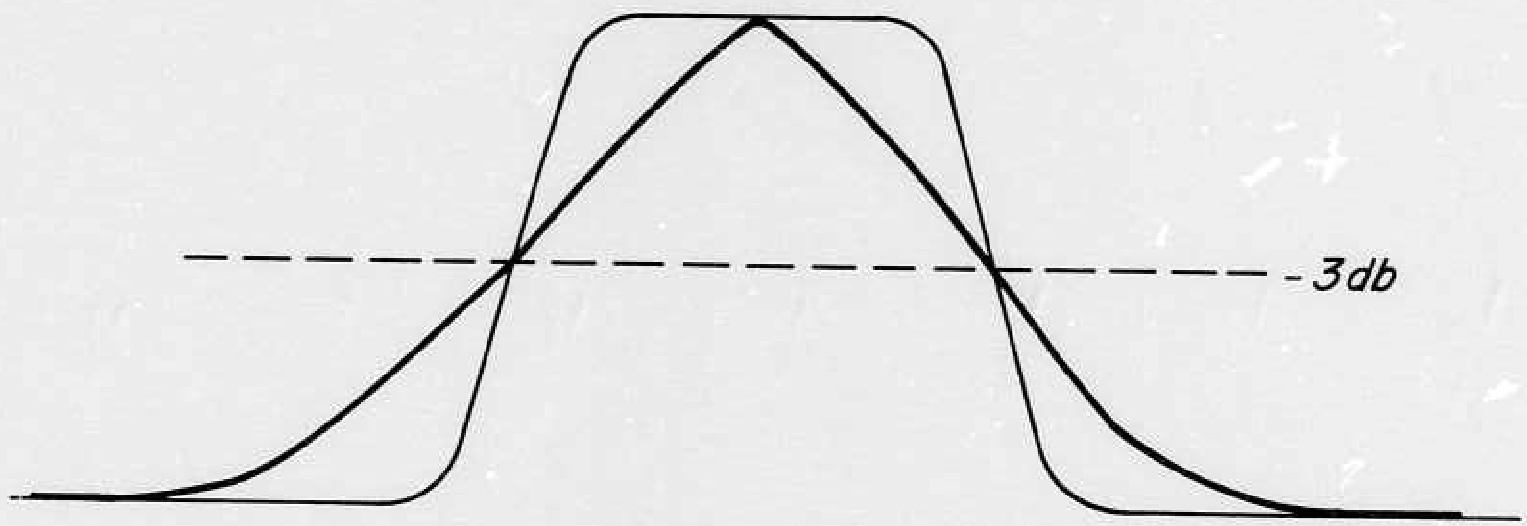


Figure 3. Two different array responses having the same width at -3 db.

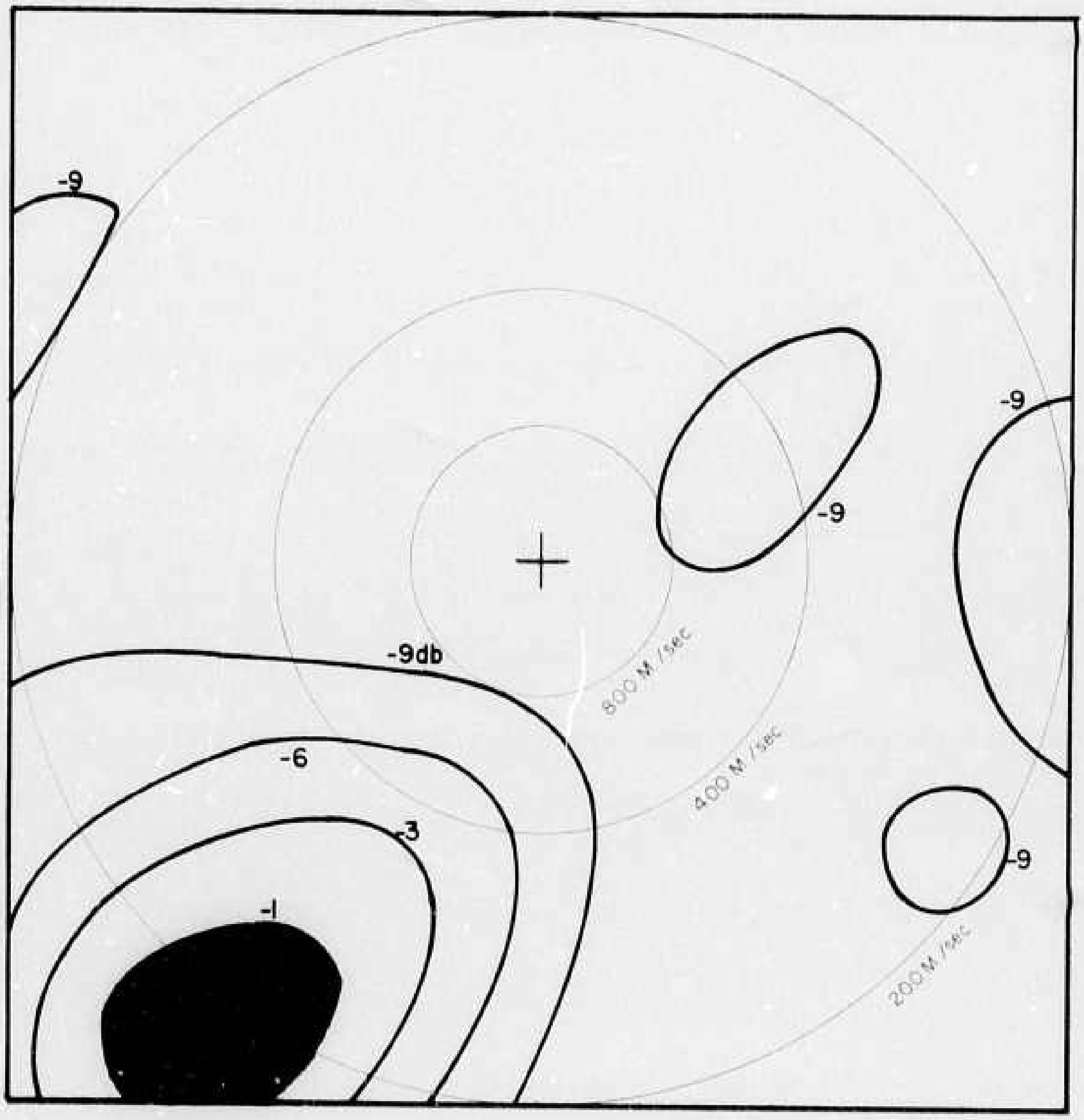


Figure 4a. Ordinary frequency-wavenumber spectra of the acoustic-gravity wave signal 128 seconds period.

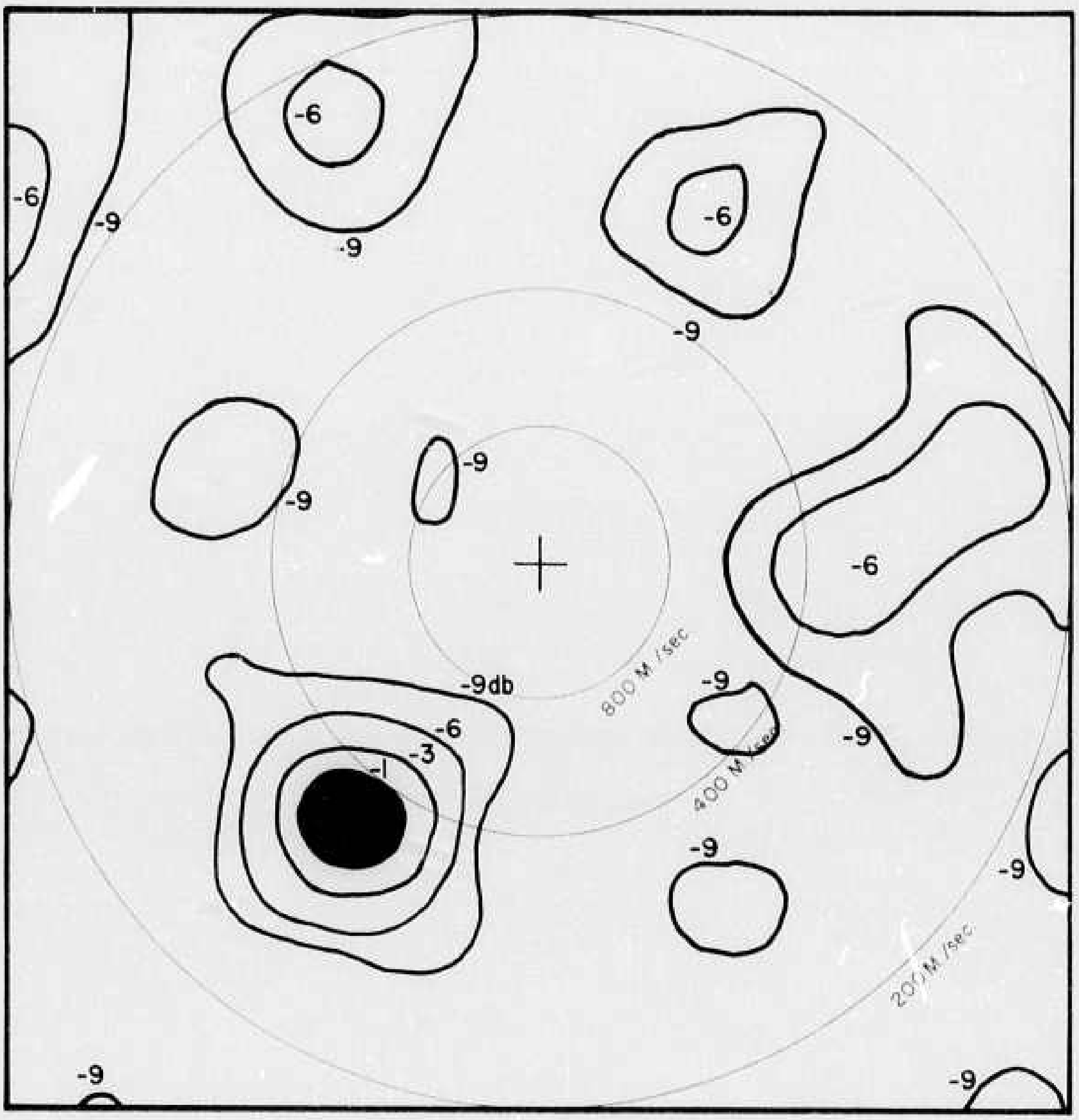


Figure 4b. Ordinary frequency-wavenumber spectra of the acoustic-gravity wave signal 64 seconds period.

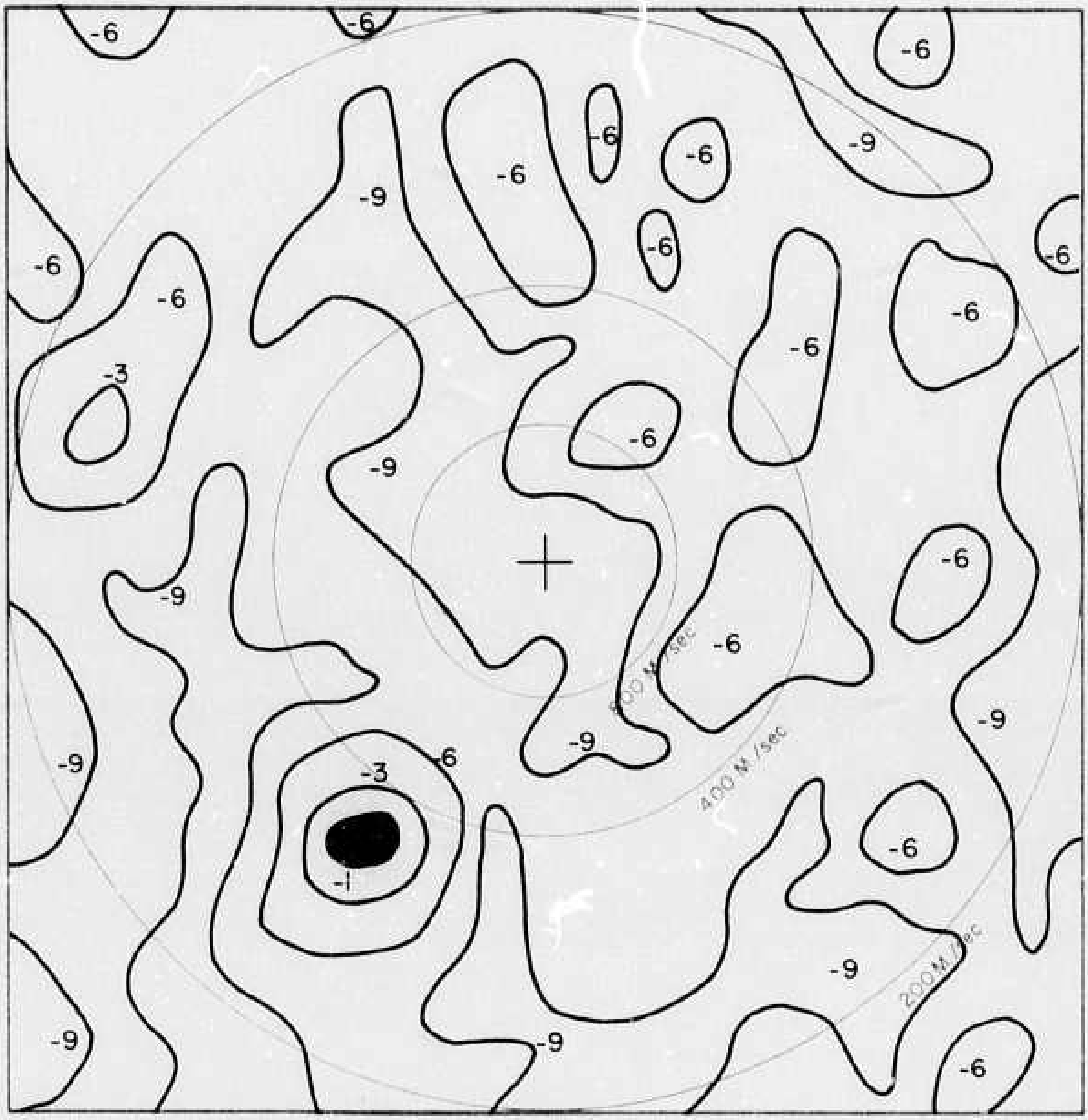


Figure 4c. Ordinary frequency-wavenumber spectra of the acoustic-gravity wave signal 43 seconds period.

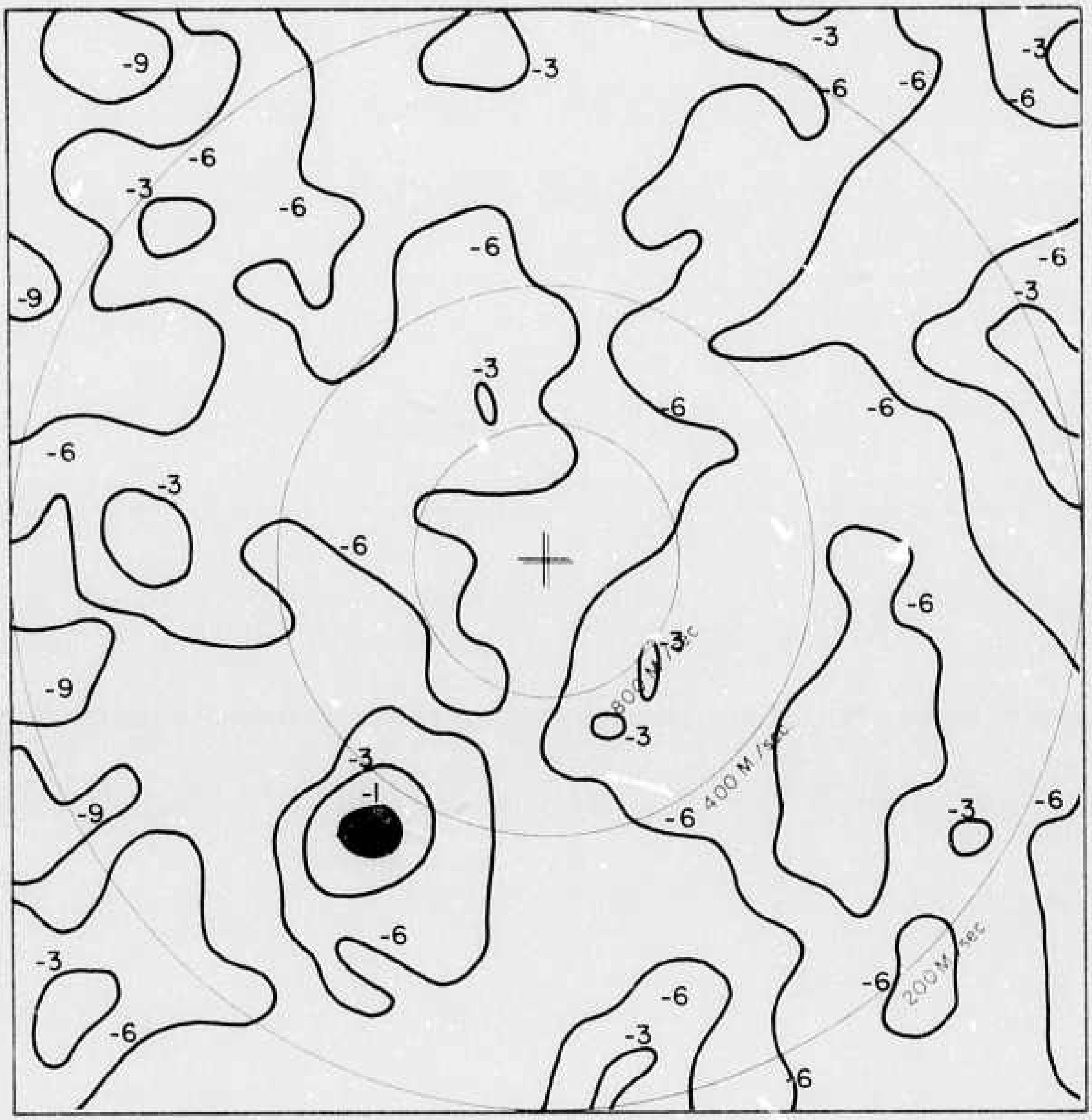


Figure 4d. Ordinary frequency-wavenumber spectra of the acoustic gravity wave signal 32 seconds period.

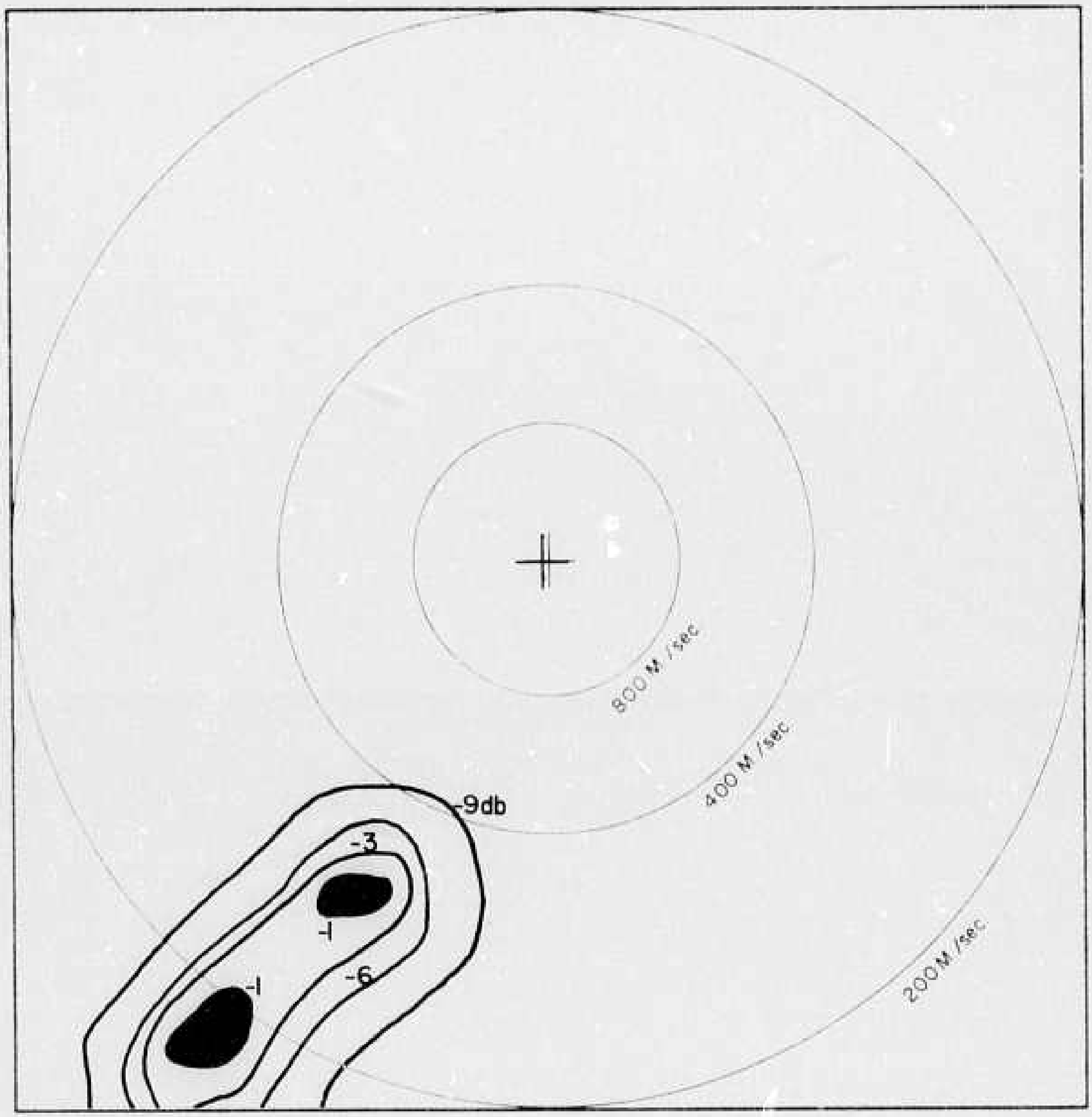


Figure 5a. High-resolution frequency-wavenumber spectrum of the acoustic-gravity wave signal 128 seconds period.

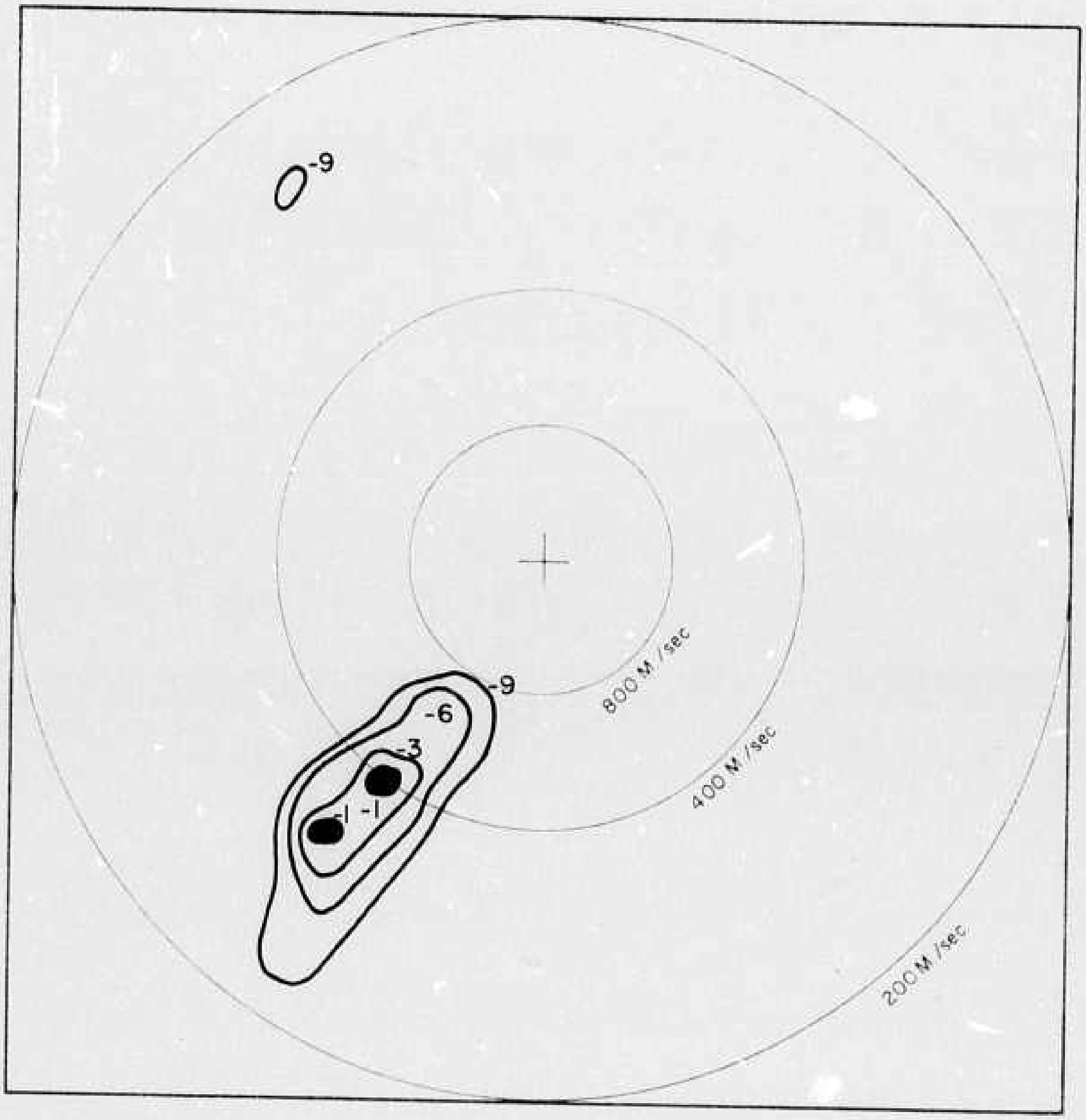


Figure 5b. High-resolution frequency-wavenumber spectrum of the acoustic-gravity wave signal 64 seconds period.

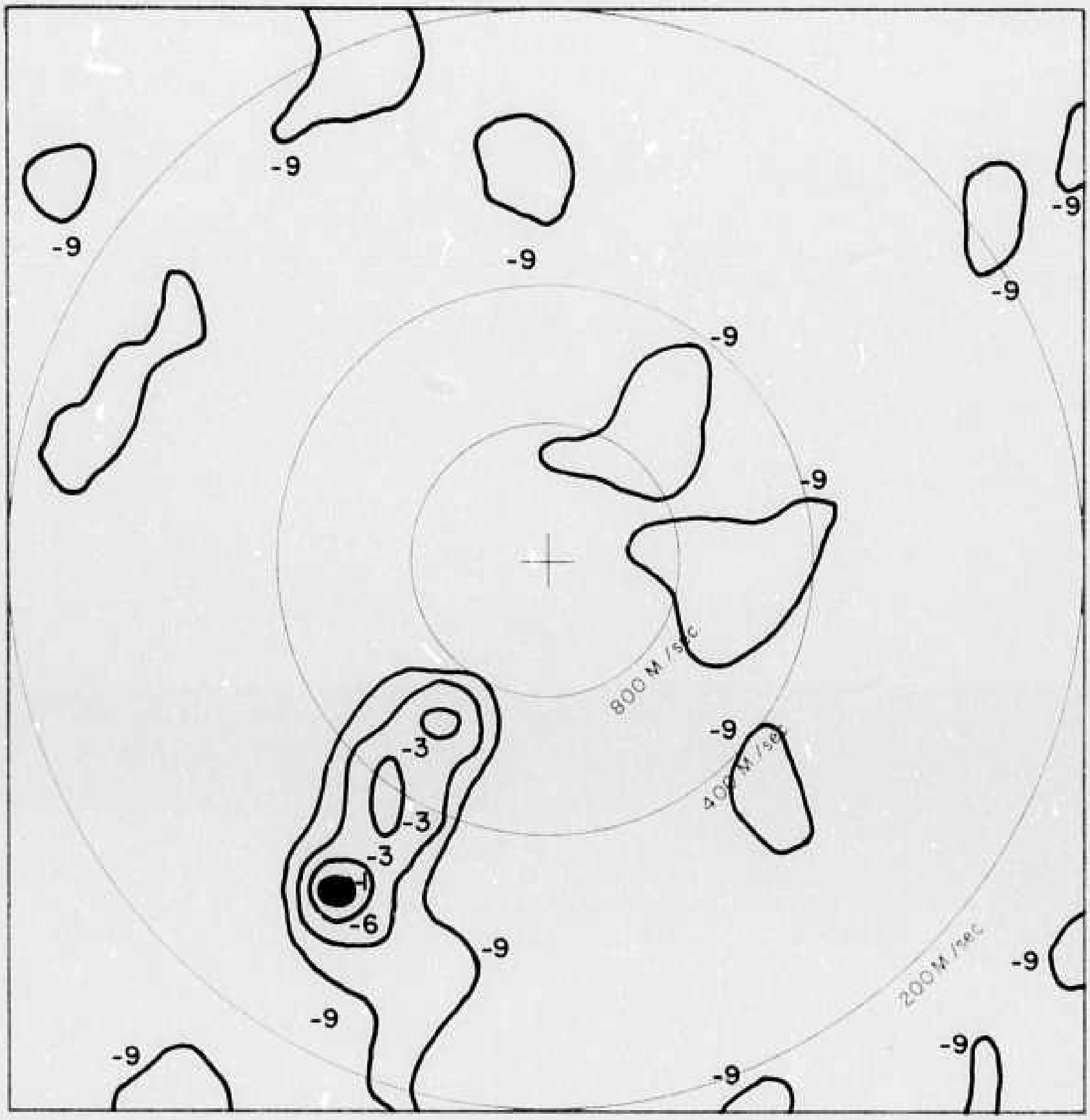


Figure 5c. High-resolution frequency-wavenumber spectrum of the acoustic-gravity wave signal 43 seconds period.

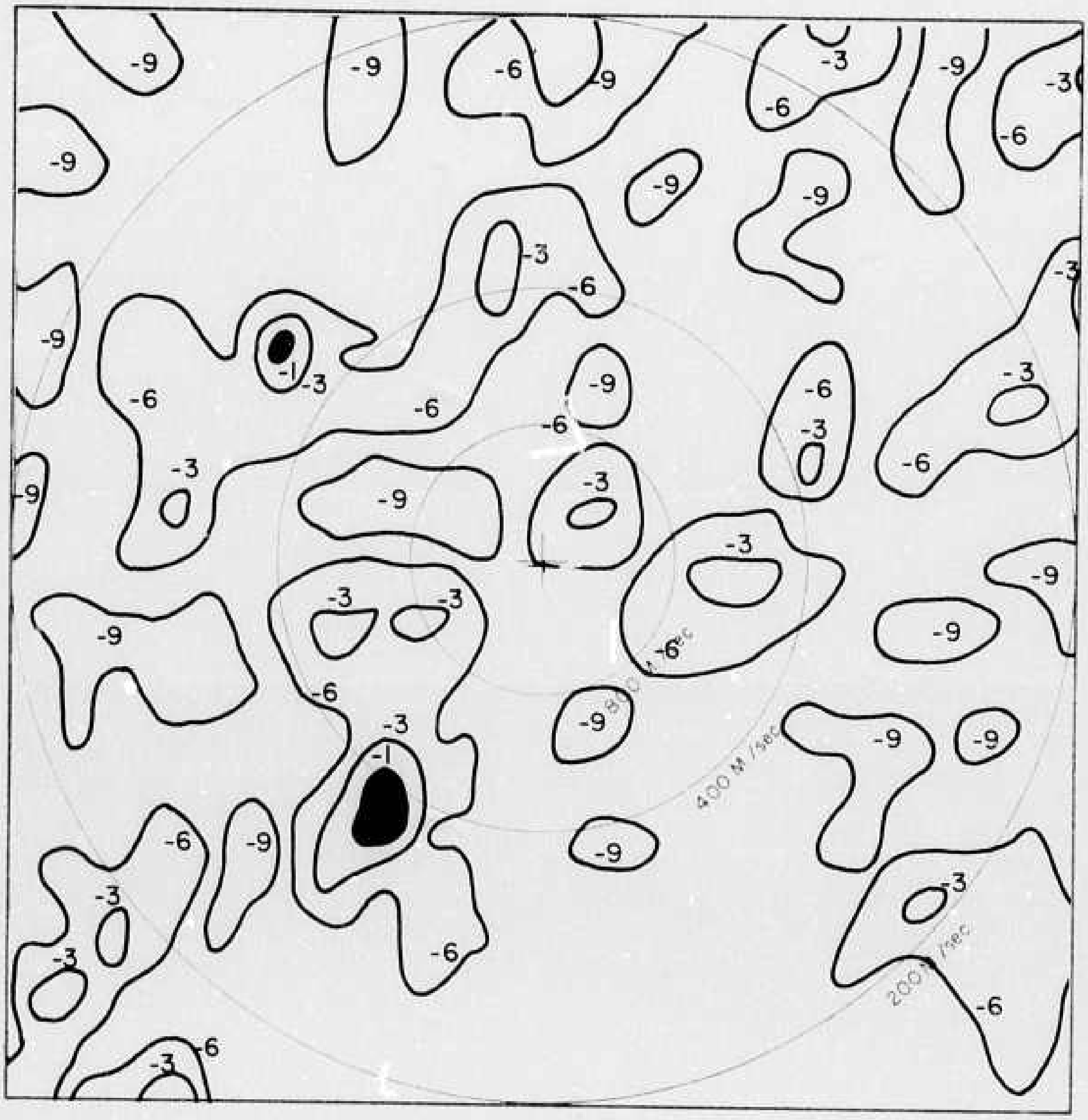


Figure 5d. High-resolution frequency-wavenumber spectrum of the acoustic-gravity wave signal 32 seconds period.

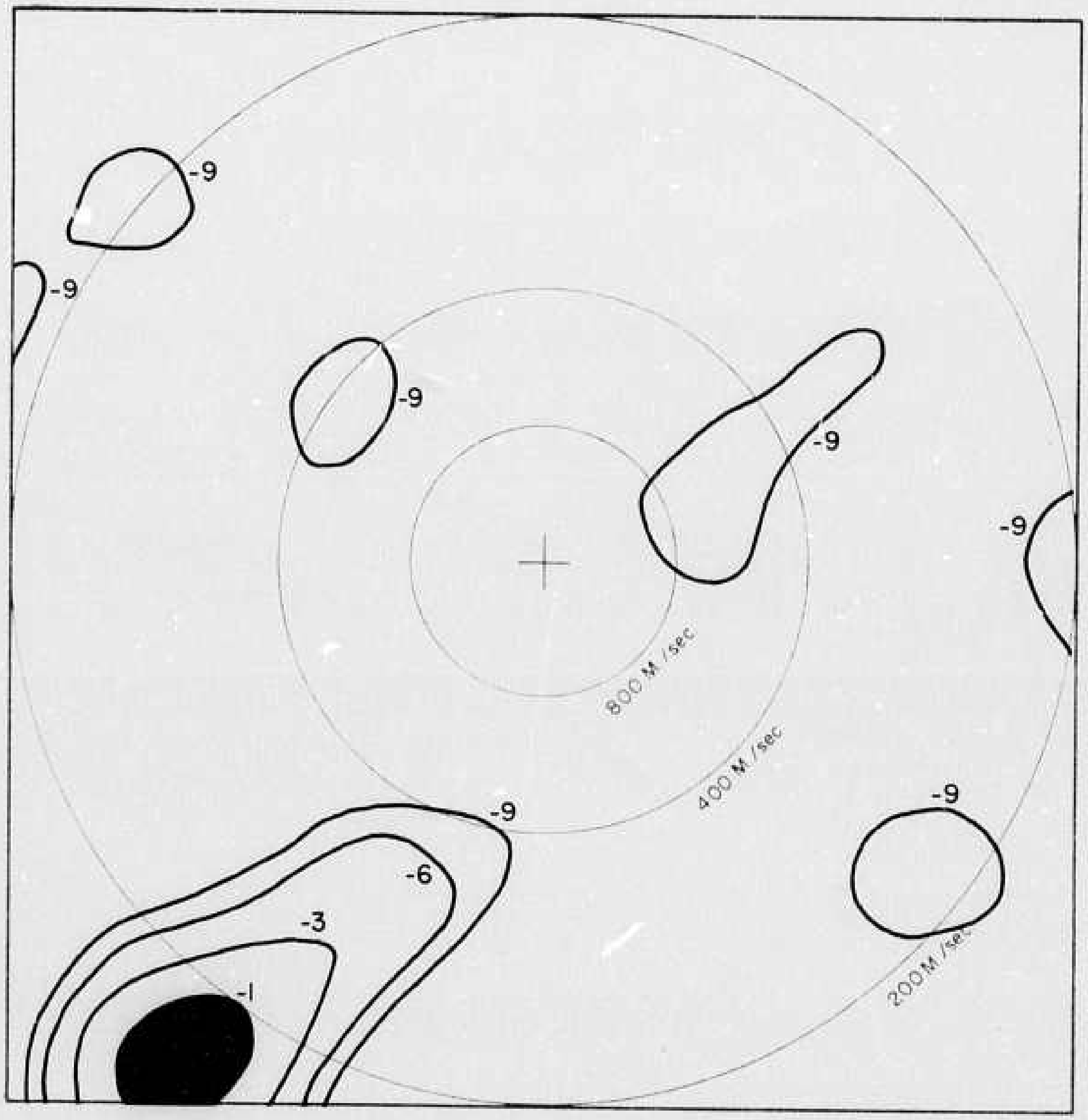


Figure 6a. Curvature frequency-wavenumber spectrum of the acoustic-gravity wave signal 128 seconds period.

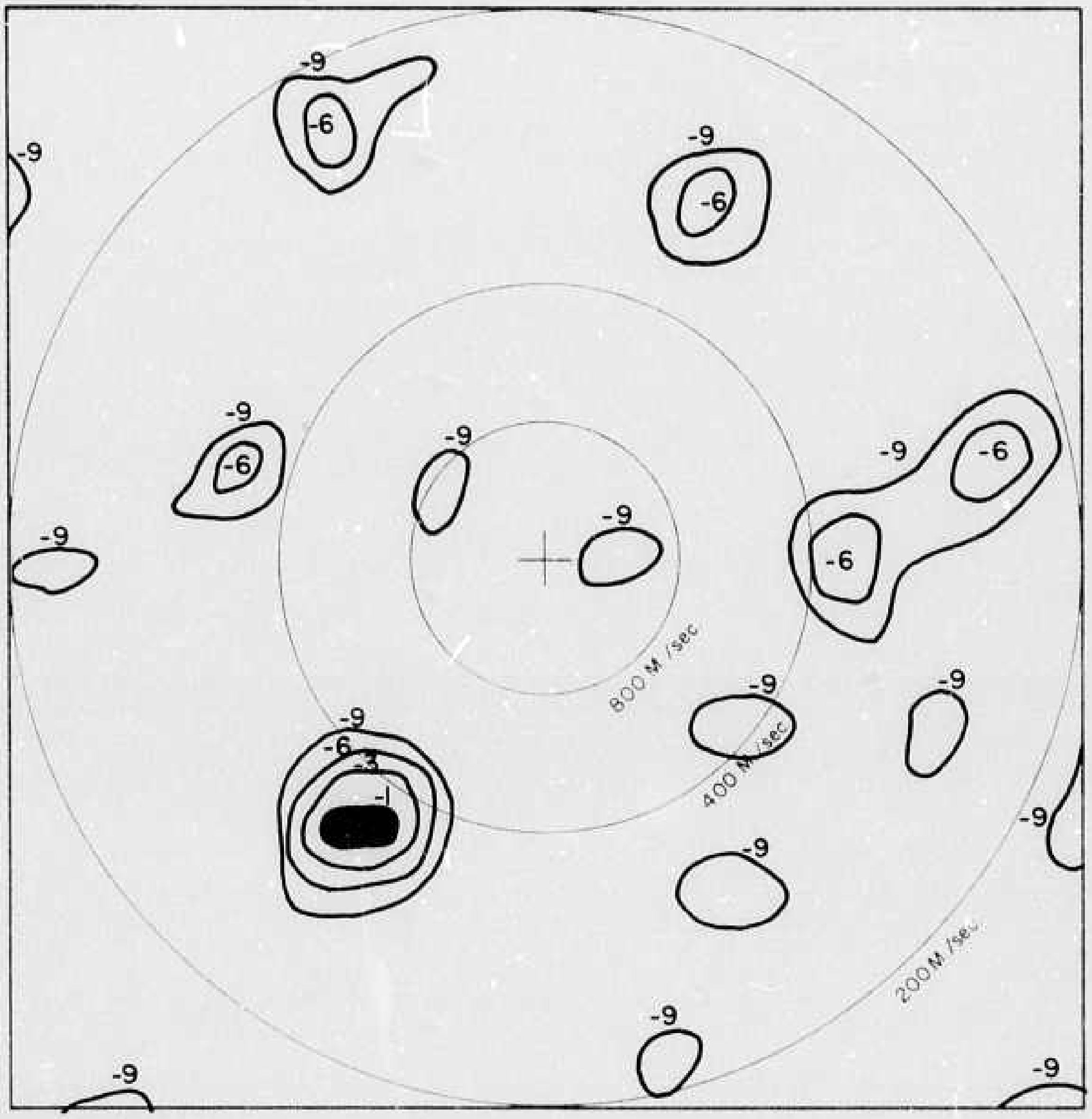


Figure 6b. Curvature frequency-wavenumber spectrum of the acoustic-gravity wave signal 64 seconds period.

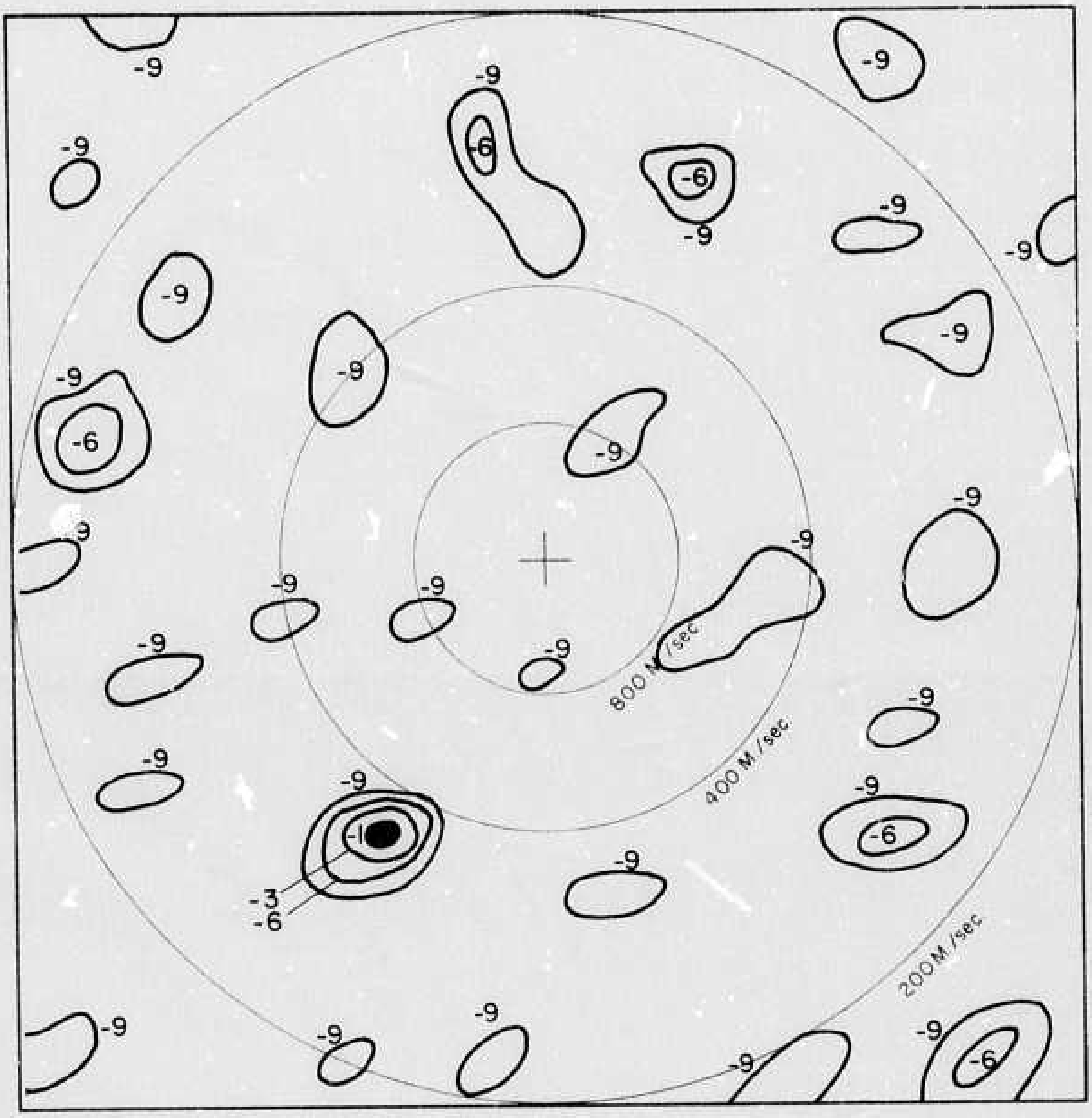


Figure 6c. Curvature frequency-wavenumber spectrum of the acoustic-gravity wave signal 43 seconds period.

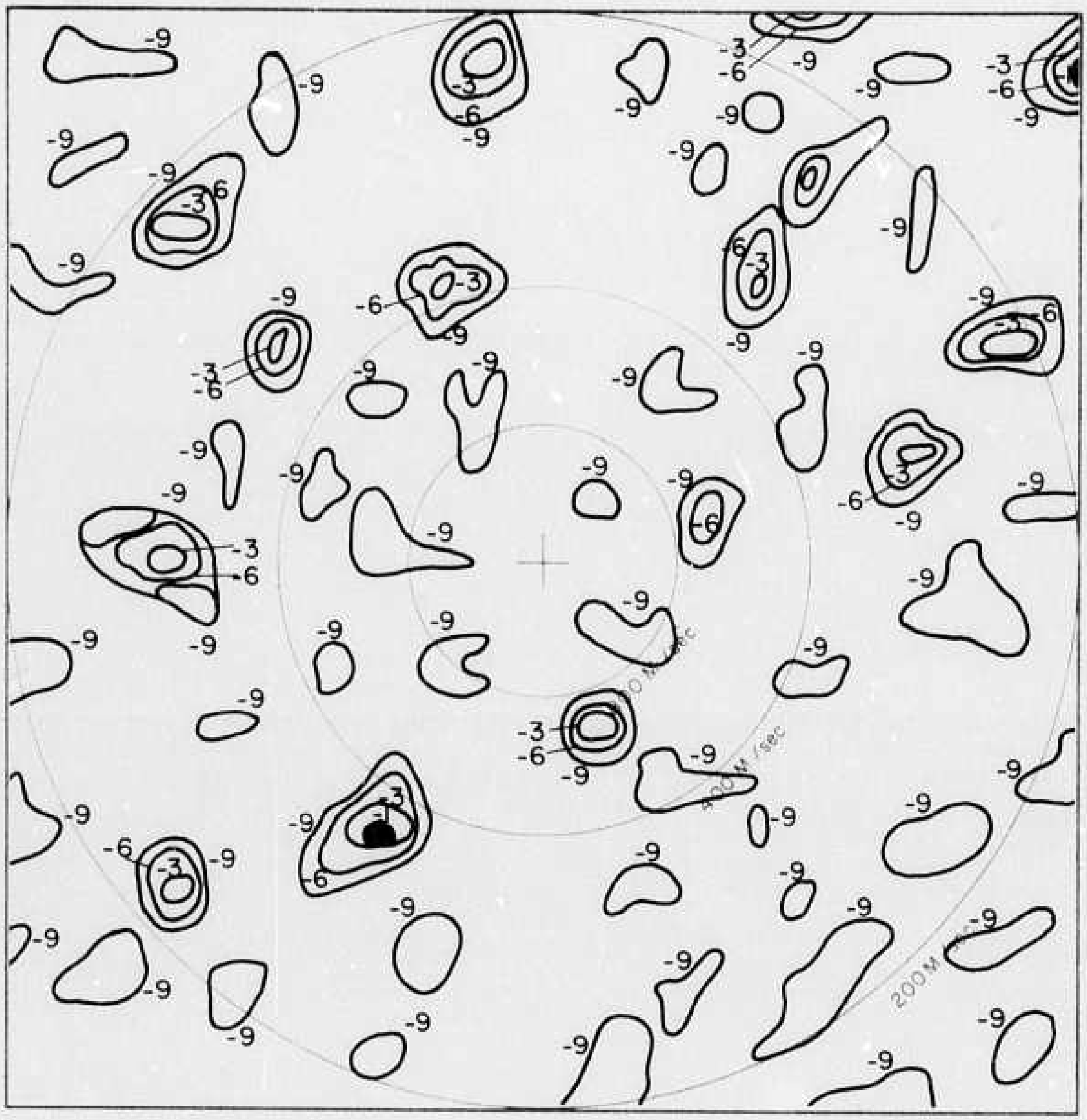


Figure 6d. Curvature frequency-wavenumber spectrum of the acoustic-gravity wave signal 32 seconds period.

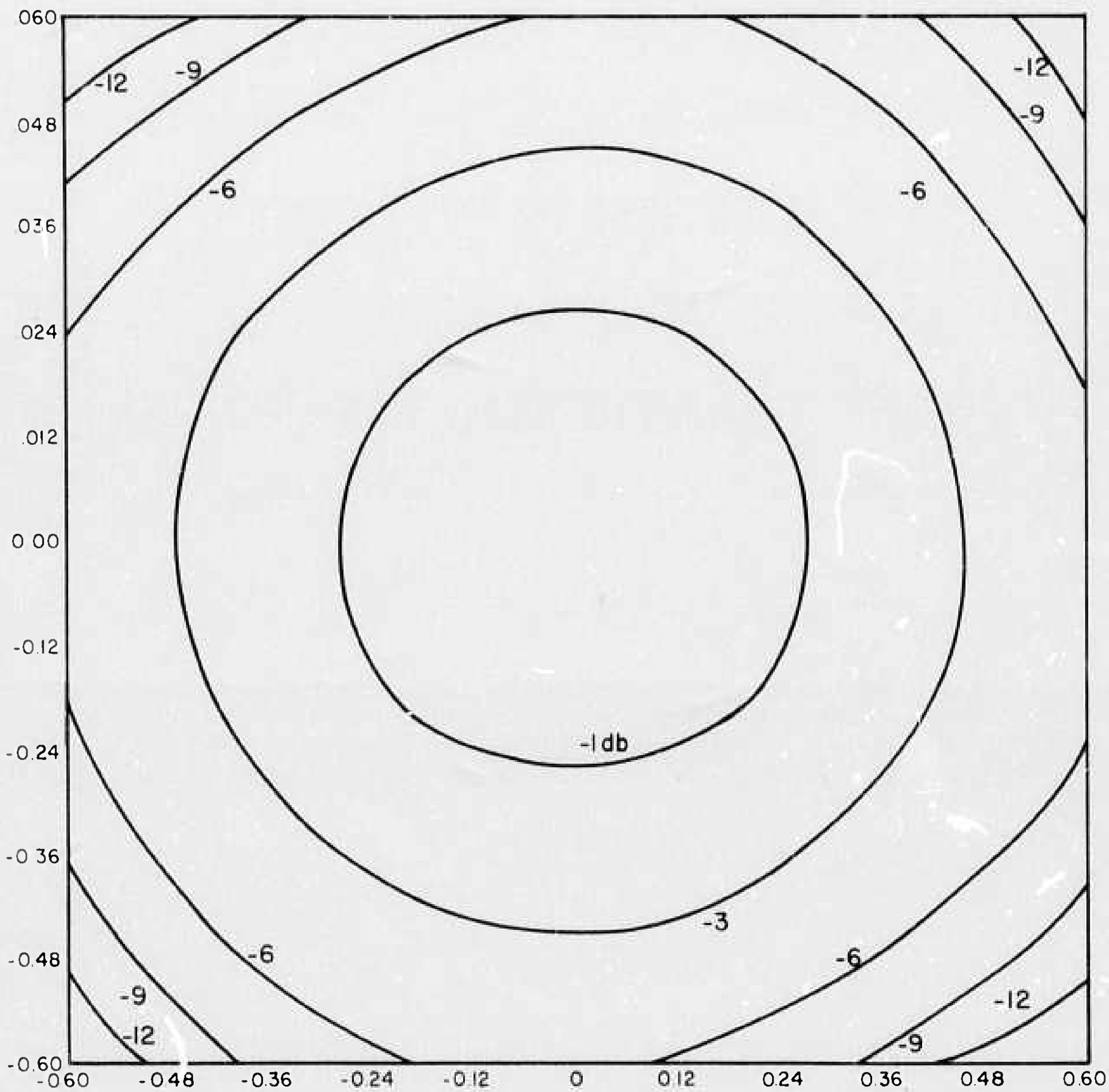


Figure 7. Huancayo array response, contoured in decibels.

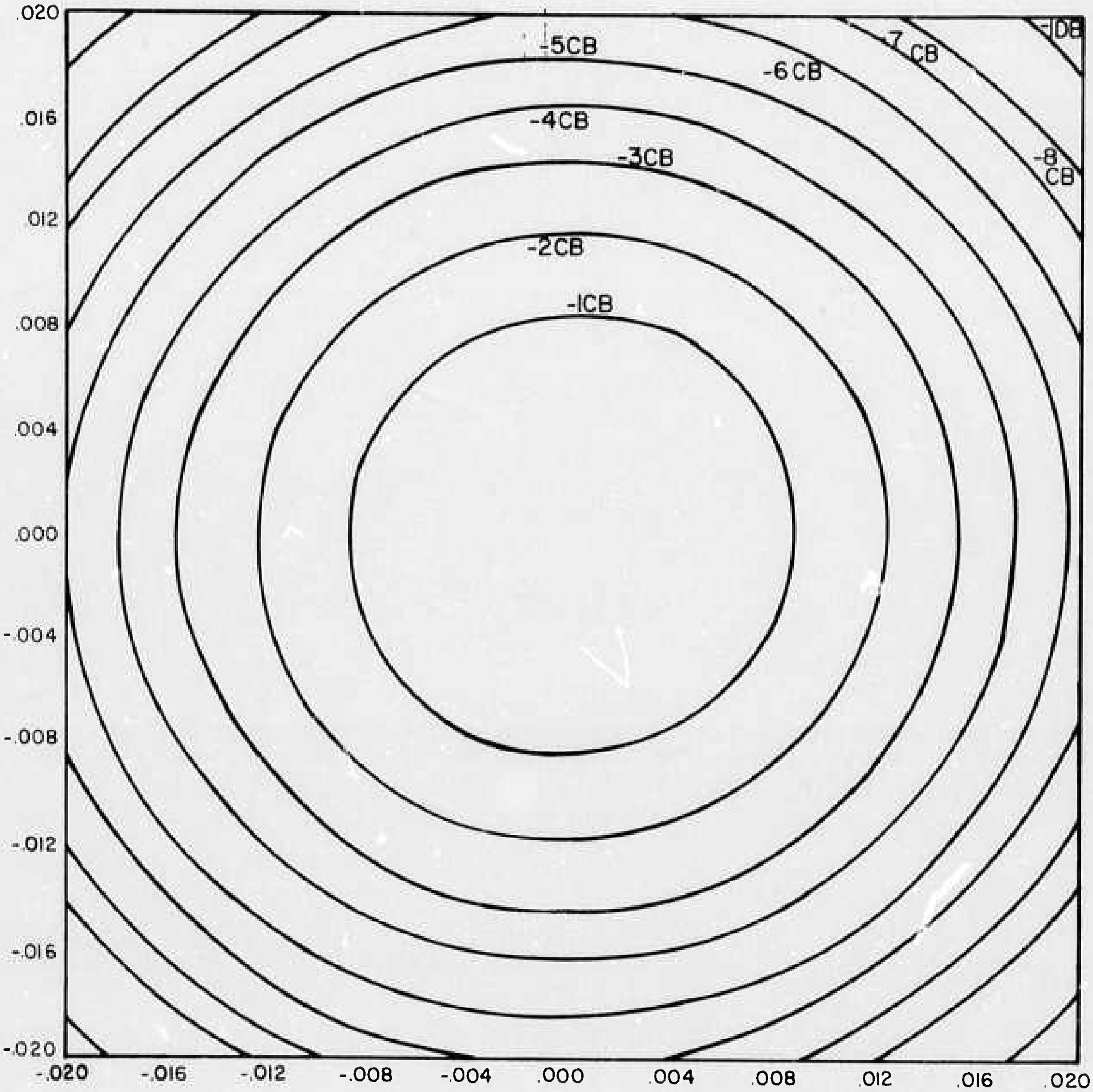


Figure 8. Huancayo array response, contoured in centibels.

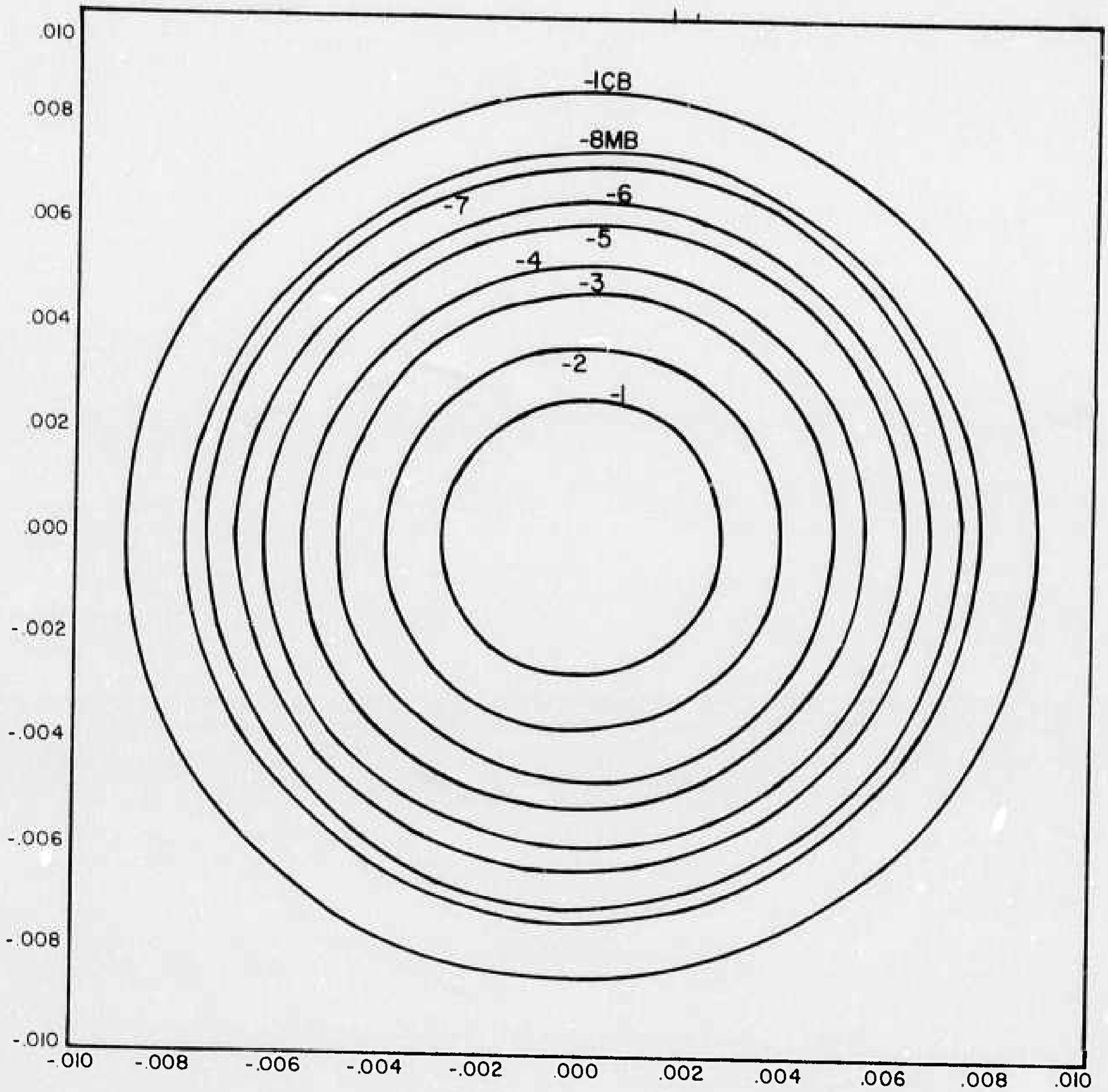
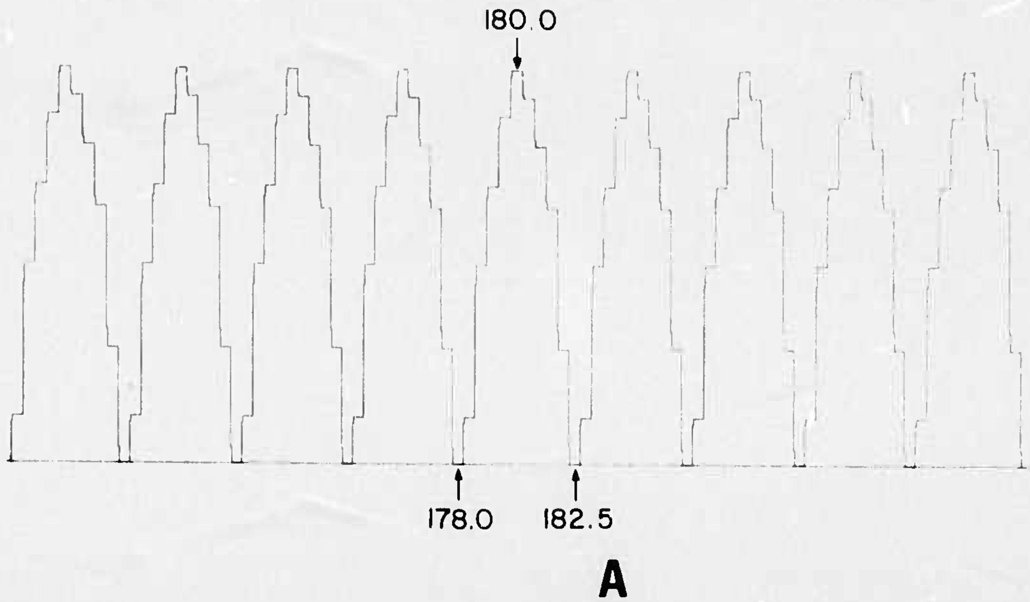


Figure 9. Huancayo array response, contoured in millibels.



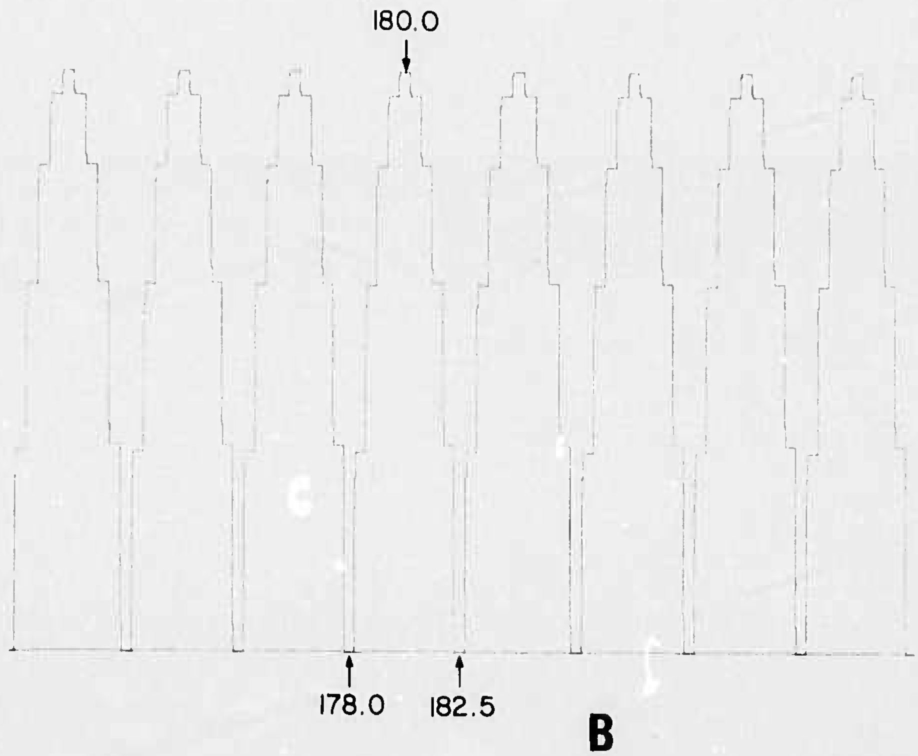


Figure 10. N4 correlator output for an input consisting of a monochromatic plane wave, 20 seconds period, velocity 300 meter/second: (a) sampling rate infinite; (b) sampling rate 4 Hz.